The Practice of Structural Equation Models (SEM)

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1. A fairy tail on simultaneous equations (with model assistance)
2. Measurement error, impact in regression analysis
3. Factor Analysis (EFA, CFA)
4. Simultaneous equations (reverse causation?)
5. Foundations of SEM
6. Models: semIV, MTMM, MIMIC, . . .

Section 1

A fairy tail on simultaneous equations (with model assistance)

Data & bivariate associations

```r
names(dat)
```

```
[1] "x0" "x1" "x2" "x3"
```

```r
dim(dat)
```

```
[1] 6000    4
```

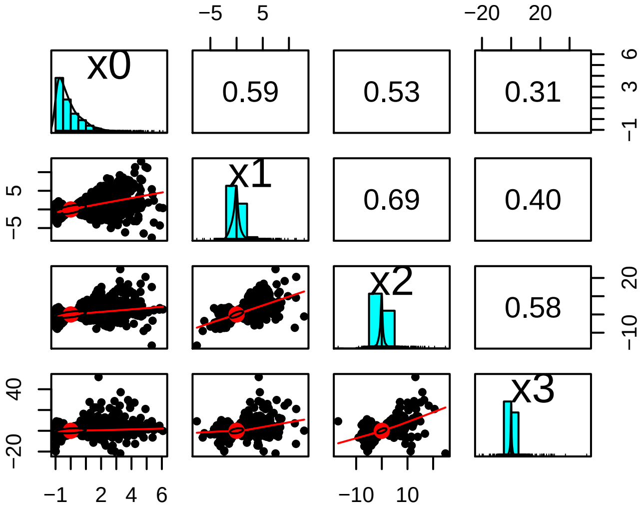
```r
head(dat,3)
```

```
           x0         x1          x2          x3
1 -0.53689183 0.18824547 -0.07486671  0.04023354
2  0.03255984 0.06865843  0.01178763  0.01408159
3 -0.44221848 0.22418833 -0.23575801 -0.14160954
```

```r
cov(dat)
```

```
          x0        x1       x2        x3
x0 1.0000000 0.7469385 0.952174 0.7721981
x1 0.7469385 1.5906099 1.579950 1.2363536
x2 0.9521740 1.5799496 3.273289 2.5708496
```

Pairs scatter & correlation plot



One equation, regression: x3 on x0, x1, x2

```
fit<- lm(x3~ x0+x1+x2, data=dat)
```

```
Call:
lm(formula = x3 ~ x0 + x1 + x2, data = dat)

Residuals:
    Min      1Q  Median      3Q     Max 
-41.218  -0.288  -0.025   0.235  41.475 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.02995    0.02585   1.158    0.247    
x0           0.04161    0.03275   1.271    0.204    
x1          -0.01993    0.03060  -0.651    0.515    
x2           0.78292    0.02021  38.733   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.002 on 5996 degrees of freedom
Multiple R-squared:  0.3352,    Adjusted R-squared:  0.3348 
F-statistic:  1008 on 3 and 5996 DF,  p-value: < 2.2e-16
```

... variables x0,x1,x2 compete to each other to explain x3. Nothing left (to explain) by variables x0 and x1, once controlling for x2. Markov view (Model), the future depends only on the recent past?. Let's check this!

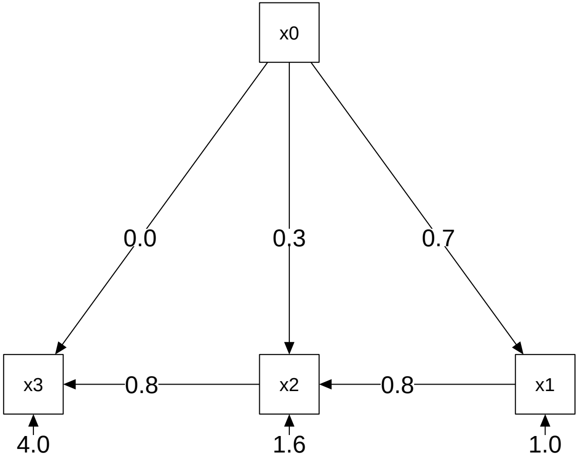
(Simultaneous) Several regressions

```r
library(lavaan); library(semPlot)

model<-"
x3~ x2+ x0;
x2~ x1+ x0;
x1~  x0
"

fit<- sem(model, estimator="MLM", data=dat)
```

Inferences and model test

```
lavaan 0.6.16 ended normally after 1 iteration

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         8

  Number of observations                          6000

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                                 0.424       0.063
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.515       0.802
  Scaling correction factor                                  6.736
    Satorra-Bentler correction

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  x3 ~
    x2                0.776    0.102    7.598    0.000
    x0                0.034    0.084    0.402    0.687
  x2 ~
    x1                0.841    0.057   14.704    0.000
    x0                0.324    0.044    7.351    0.000
  x1 ~
    x0                0.747    0.036   20.959    0.000

Variances:
```

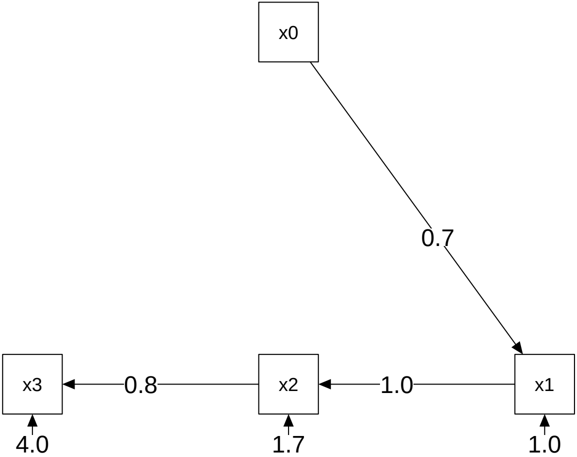
path diagram of the fitted model



Markovian model (exact ?)

```
model<-"
x3~ x2 ;
x2~ x1 ;
x1~  x0"
fit<- sem(model, estimator="MLM", data=dat)
```

Path diagram (exact Markov)



Estimates and model test

```
lavaan 0.6.16 ended normally after 1 iteration

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         6

  Number of observations                          6000

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               246.307      44.231
  Degrees of freedom                                 3           3
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  5.569
    Satorra-Bentler correction

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  x3 ~
    x2                0.785    0.085    9.221    0.000
  x2 ~
    x1                0.993    0.050   19.750    0.000
  x1 ~
    x0                0.747    0.036   20.959    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x3                4.008    0.514    7.796    0.000
```

Missfit of model?

```
modificationindices(fit,sort=TRUE, power=TRUE)[,-c(6:7)]
```

```
   lhs op rhs      mi    epc     ncp power decision
16  x1  ~  x2 239.734 -0.263  34.725 1.000  *epc:m*
10  x2 ~~  x1 239.734 -0.448  11.964 0.933  *epc:m*
18  x0  ~  x2 239.734  0.190  66.376 1.000  *epc:m*
14  x2  ~  x0 239.734  0.324  22.862 0.998  *epc:m*
17  x0  ~  x3  53.630  0.047 241.009 1.000   epc:nm
15  x1  ~  x3  39.615 -0.047 181.822 1.000   epc:nm
9   x3 ~~  x1   1.088 -0.033   9.986 0.885     (nm)
12  x3  ~  x0   1.068  0.029  12.451 0.942     (nm)
8   x3 ~~  x2   0.037  0.009   4.213 0.537      (i)
11  x3  ~  x1   0.037 -0.005  12.394 0.941     (nm)
13  x2  ~  x3   0.037  0.002  67.674 1.000     (nm)
```

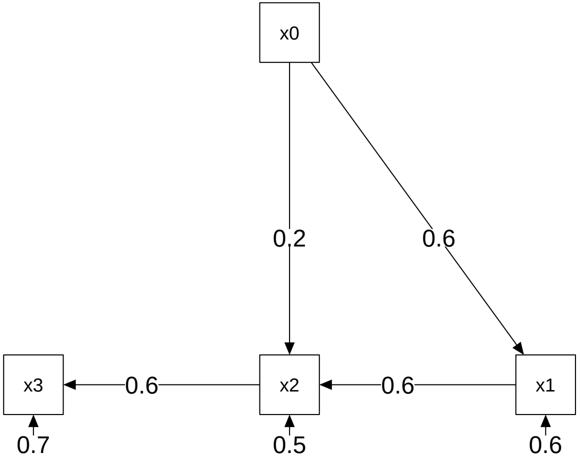
Approximate Markovian model

```r
library(lavaan); library(semPlot)
model<-"x3~ x2 ; x2~ x1 ;  x1~  x0;  x2 ~ x0"
fit<- sem(model, estimator="MLM", data=dat)
```

Final Model

```
lavaan 0.6.16 ended normally after 1 iteration

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         7

  Number of observations                          6000

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                                 1.653       0.228
  Degrees of freedom                                 2           2
  P-value (Chi-square)                           0.438       0.892
  Scaling correction factor                                  7.241
    Satorra-Bentler correction

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Structured

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  x3 ~
    x2                0.785    0.085    9.221    0.000
  x2 ~
    x1                0.841    0.057   14.704    0.000
  x1 ~
    x0                0.747    0.036   20.959    0.000
  x2 ~
    x0                0.324    0.044    7.351    0.000

Variances:
```

path diagram: std



Longitudinal data: Markovian model with mease

Markovian with no-mease, had a poor fit: chi2=102.351, df=3

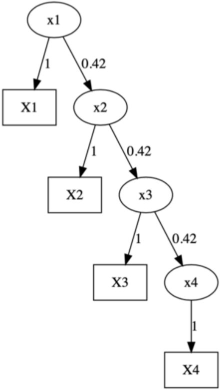
No account for measurement error, model modified to fit

```
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  X2 ~
    X1                0.174    0.005   34.306    0.000
  X3 ~
    X2                0.225    0.008   28.296    0.000
  X4 ~
    X3                0.234    0.012   18.717    0.000
    X2                0.045    0.007    6.005    0.000
  X3 ~
    X1                0.039    0.005    7.857    0.000
```

```
Model Test User Model:
                                                Standard      Robust
  Test Statistic                                  11.847       3.338
  Degrees of freedom                                   1           1
  P-value (Chi-square)                             0.001       0.068
  Scaling correction factor                                    3.549
       Satorra-Bentler correction
```

Longitudinal data: markovian model on LV

Londitudinal data with account for measurement error (using the simplex model)



Markovian model (Simplex) is accepted. (SB-scaled) Chi2 = 1.749, df=4,

. . . what is a model?

1. . . . for a statistician, it is a likelihood, a known distribution for (univariate or) multivariate data object, fully specified except for a set of parameters.
2. For a Bayesian, idem as 1., with the extra of *prior distribution* (multivariate) for the set of parameters
3. In SEM: a model is a set of (simultaneous) regression equations expressing prior knowledge of interrelations among observable and (possibly) latent variables, plus prior assumptions of conditional independence (or just uncorrelation) among variables. Our aim is distribution-free inferences both on estimates and fit of the model.
4. The sample var-cov matrix of observable variables is a sufficient statistic for estimates. Distribution-free inferences (se and model test) require a matrix of fourth-order moments. Non-linear models also require higher-order moments for consistent estimates.

One-regression: mediators and confounders ?

We have data on y, x_1, x_2. The true model is

$$y = 0 + \gamma_1 x_1 + \gamma_2 x_2 + e$$

where e is a random normal distribution of mean 0 variance 1. We do not have at hand x_2, and we estimate the model without this variable. Note that we can write:

$$y = \alpha + \beta x_1 + u$$

, where

$$u \gamma_2 * x_2 + e$$

is possibly correlated with y (when $\gamma_2 \neq 0$)

Behavioural equation:

$$y = \alpha + \beta x_1 + u$$

where u possibly correlated with x_1. **OLS Regression (Predictive):**

$$y = \alpha + \beta x_1 + e$$

where e has mean 0 and is uncorrelated with x_1. The *alpha* and β are not the same in the two equations.

Biass of estimates caused by confounding and mediation

```r
# x2 mediator
fit<- lm(y~ x1)
## x2 is mediator
fit$coefficients
```

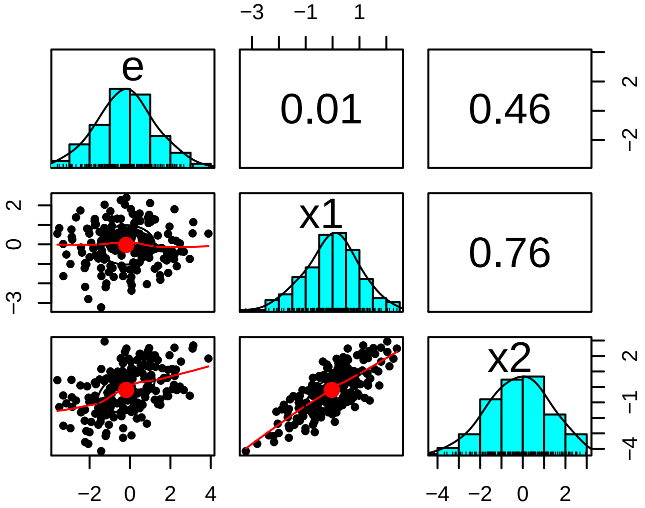
```
(Intercept)          x1
 0.01533264  1.98101341
```

```r
e<- fit$residuals


# x1 confounder
fit<- lm(y~ x2)
fit$coefficients
```

```
(Intercept)          x2
 0.01801253  1.49471298
```

We read, 1.9810 is the increase on y when x1 increases one unit *ceteris paribus* nothing ! .

pairs plot and correlations



uncorrelation of residuals with x_1, but correlation of residuals with x_2

Behavioural (SEM) regression of y on x1 (+ x2 mediator)

```
  lhs op rhs   est    se       z pvalue
1   y  ~  x1 1.981 0.018 107.731      0
2  x1 ~~  x2 0.971 0.022  44.378      0
3   y ~~  x2 1.000 0.022  44.723      0
4   y ~~   y 2.000 0.037  54.772      0
5  x1 ~~  x1 0.986 0.018  54.772      0
6  x2 ~~  x2 1.955 0.036  54.772      0

(Intercept)          x1
 0.01533264  1.98101341
```

Problem fixed by multiple reg (when potential mediators or confounding are observed)

	lhs	op	rhs	est	se	z	pvalue	ci.lower	ci.upper
1	y	~	x1	0.996	0.018	54.805	0	0.961	1.032
2	y	~	x2	1.000	0.013	77.468	0	0.975	1.025
3	x1	~~	x2	0.971	0.022	44.378	0	0.928	1.013
4	y	~~	y	1.000	0.018	54.772	0	0.964	1.036
5	x1	~~	x1	0.986	0.018	54.772	0	0.951	1.021
6	x2	~~	x2	1.955	0.036	54.772	0	1.885	2.025

```
(Intercept)          x1          x2
  0.0203646   0.9964539   1.0001305
```

The effect on y of unit increase of x1 is 1.023 *ceteris paribus* x2. I know that 1 is the true value (population value), I generated the data.

Section 2

Measurement error, impact in regression analysis

Reliability of X:

$$X = x + \epsilon$$

$$k_X = \frac{\sigma_x^2}{\sigma_X^2}$$

The value of k_X is known as the *reliability coefficient* of X, for measuring the true x. Note that $\sigma_\epsilon^2 = (1 - k)\sigma_X^2$.

For simple linear regression the effect is an attenuation of the regression coefficient. This is known as *attenuation bias*. In more complicated settings, assessing the direction of the bias due to mease is more complex.

Measurement error and endogeneity in the regression

Two simultaneous equations in action:

$$Y = \alpha + \beta x + U$$

and

$$X = x + \epsilon$$

Thus

$$\begin{aligned} Y &= \alpha + \beta(X - \epsilon) + U \\ &= \alpha + \beta X + U^* \end{aligned}$$

where $U^* = U - \beta\epsilon$. Note that

$$\text{cor}\,(X, U^*) = \text{cor}\,(x + \epsilon, U - \beta\epsilon) = -\beta\sigma_\epsilon^2 \neq 0$$

except when β and/or σ_ϵ^2 are zero.

Fuller's reliability table

Table 1.1.1. of Fuller (1987, p. 8) shows estimates of reliability coefficients for a number of socioeconomic variables. Repeated interview conducted by the United States Bureau of the Census. Comparison of responses in the 1970 Census with the same data collected in the Current Population Survey. In survey sampling, the measurement error in data collected from human respondents is uaually called *response error*

Variable	k
Sex	.98
Age	.99
Age (45-49)(0-1)	.92
Education	.88
Income	.85
Unemployed	.77
Poverty status	.58

Toy example: x = True Alcohol Intake (Tintake), y = Driver Reaction Time (DRT) , X = Observable Alcohol Intake (Ointake)

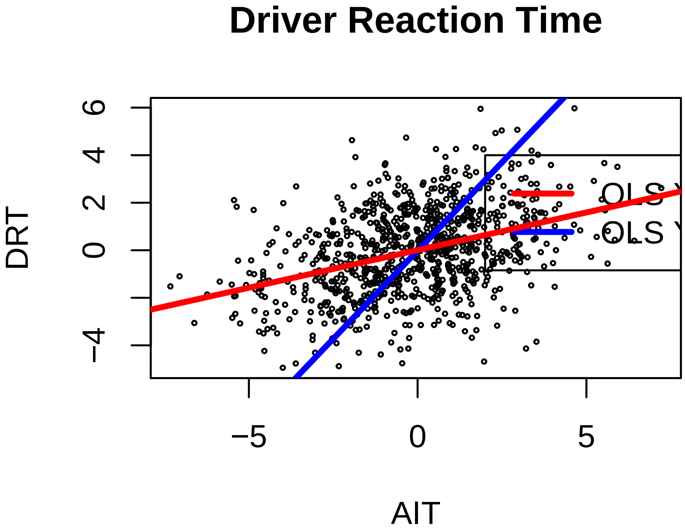
The reliability is the ratio of two variances

$$k = \frac{\mathrm{var}\,(\mathrm{Tintake})}{\mathrm{var}\,(\mathrm{Ointake})} = 1 - \frac{\mathrm{var}\,(\mathrm{error})}{\mathrm{var}\,(\mathrm{Ointake})}$$

is the so-called reliability of Ointake. The reliability of Ointake is likely to be $k \neq 1$.

When $k_X < 1$

OLS regression estimator is not consistent for the slope of the regression equation



Data of Y with two indicators of true intake: Ointake1, Ointake2

```
[1] 862   3

  Resp Ointake1 Ointake2
1 9.89     8.76     9.59
2 9.70     5.45     8.34
3 9.91    11.46     9.46

[1] "Resp"     "Ointake1" "Ointake2"

[1] 862   3

   Resp Ointake1 Ointake2
1  9.89     8.76     9.59
2  9.70     5.45     8.34
3  9.91    11.46     9.46
4 10.14    11.57    11.08
5 10.26    12.10    11.34
6  9.96    10.38     9.52

          Resp Ointake1 Ointake2
Resp     0.033    0.159    0.157
Ointake1 0.159    5.019    0.984
Ointake2 0.157    0.984    1.214
```

OLS vs. SEM: errors-in-variables regression (estimates)

```
## OLS regression:
summary(lm(Resp~ Ointake1, data=data))$coefficients[2,]
```

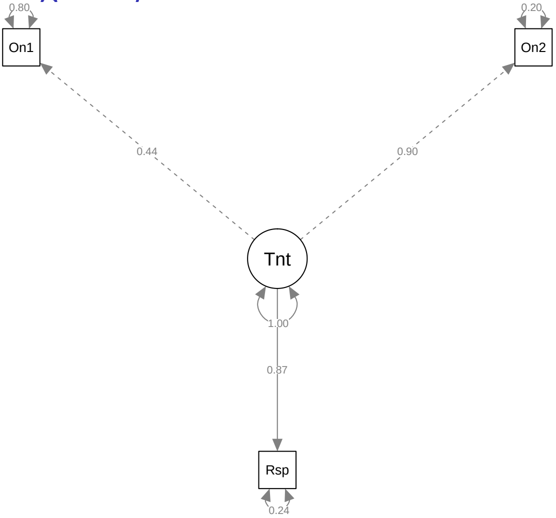
```
    Estimate   Std. Error      t value     Pr(>|t|)
3.167092e-02 2.545920e-03 1.243987e+01 8.710154e-33
```

```
library(lavaan)
#######################################
pdmodel2 <- "
        # latent variable definitions
       Tintake =~  1*Ointake1 + 1*Ointake2
        # regression
        Resp ~ Tintake
            "
mlfit2 <-  lavaan(pdmodel2, data = data, auto.var = TRUE)
parameterestimates(mlfit2, ci=FALSE)
```

```
      lhs op      rhs   est    se      z pvalue
1  Tintake =~ Ointake1 1.000 0.000     NA     NA
2  Tintake =~ Ointake2 1.000 0.000     NA     NA
3     Resp  ~  Tintake 0.161 0.009 18.783      0
4 Ointake1 ~~ Ointake1 4.019 0.199 20.163      0
5 Ointake2 ~~ Ointake2 0.240 0.049  4.920      0
6     Resp ~~     Resp 0.008 0.001  5.955      0
7  Tintake ~~  Tintake 0.974 0.072 13.454      0
```

By accounting for measurement error we have increased the significance of the effect of intake on the response. The same model be fitted equating the error variances, however it would show a missfit.

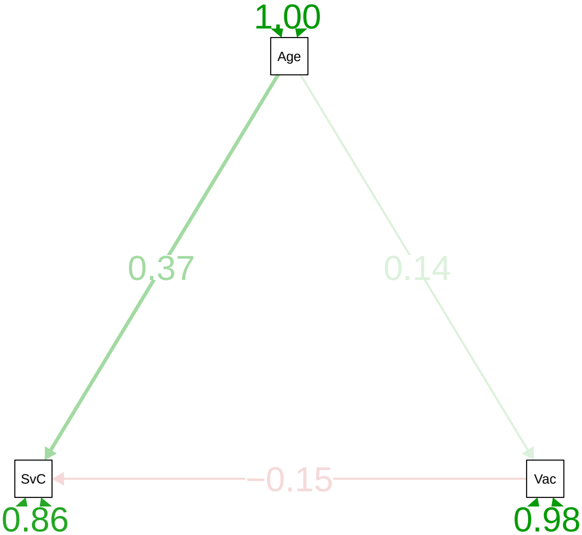
SEM (lavaan): errors-in-variables regression (std path diagram.)



With the std solution, we see the reliability (k) of the two indicators. ## Covid data (using polychoric)

Path diagram

```
semPaths(fit, what = "std", edge.label.cex = 2)
```



Section 3

Factor Analysis (EFA, CFA)

Single-Factor (correlation matrix)

Spearman, 1904

Variables

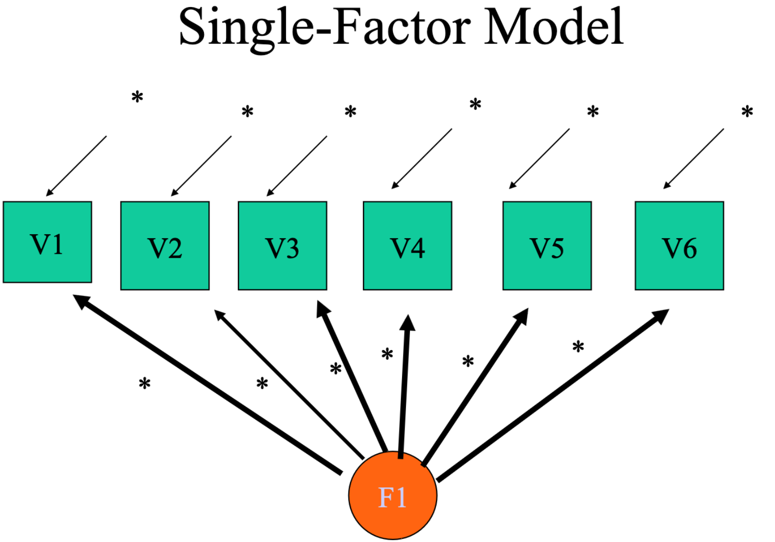
```
CLASSIC   = V1
FRENCH    = V2
ENGLISH   = V3
MATH      = V4
DISCRIM   = V5
MUSIC     = V6
```

Correlation matrix

1
.83 1
.78 .67 1
.70 .64 .64 1
.66 .65 .54 .45 1
.63 .57 .51 .51 .40 1

cases = 23;

Single Factor: path model



Factor Models (cont.)

NT analysis

RESIDUAL COVARIANCE MATRIX (S-SIGMA) :

		CLASSIC V 1	FRENCH V 2	ENGLISH V 3	MATH V 4	DISCRIM V 5	MUSIC V 6
CLASSIC	V 1	0.000					
FRENCH	V 2	-0.001	0.000				
ENGLISH	V 3	0.005	-0.029	0.000			
MATH	V 4	-0.006	0.003	0.046	0.000		
DISCRIM	V 5	-0.001	0.054	-0.015	-0.056	0.000	
MUSIC	V 6	0.003	0.005	-0.017	0.030	-0.049	0.000

CHI-SQUARE = 1.663 BASED ON 9 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS 0.99575
THE NORMAL THEORY RLS CHI-SQUARE FOR THIS ML SOLUTION IS 1.648

Two-Factor Model

Data of Lawley and Maxwell

M0:

```
/TITLE
 Lawley and Maxwell data
/SPECIFICATIONS
  CAS=220; VAR=6; ME=ML;
/LABEL
  v1 =Gaelic;
  v2 = English;
v3 = Histo;
v4 =aritm;
v5 =Algebra;
v6 =Geometry;
/EQUATIONS
V1= *F1 + E1;
V2= *F1 + E2;
V3= *F1 + E3;
V4= *F1 + E4;
V5= *F1 + E5;
V6= *F1 + E6;
/VARIANCES
  F1 = 1;   E1 TO E6 = *;
/COVARIANCES
/MATRIX
1 .439 .410 .288 .329 .248
.439 1 .351 .354 .320 .329
.410 .351 1 .164 .190 .181
.288 .354 .164 1 .595 .470
.329 .320 .190 .595 1 .464
.248 .329 .181 .470 .464 1
/END
```

M1:

```
/EQUATIONS
V1= *F1 + E1;
V2= *F1 + E2;
V3= *F1 + E3;
V4= *F2 + E4;
V5= *F2 + E5;
V6= *F2 + E6;
/VARIANCES
  F1 = 1;  F2=1;  E1 TO E6 = *;
/COVARIANCES
 F1, F2 = *;
```

```
GAELIC  =V1  =     .687*F1     + 1.000 E1
                   .076
                  9.079
ENGLISH =V2  =     .672*F1     + 1.000 E2
                   .076
                  8.896
HISTO   =V3  =     .533*F1     + 1.000 E3
                   .076
                  7.047
ARITM   =V4  =     .766*F2     + 1.000 E4
                   .067
                 11.379
ALGEBRA =V5  =     .768*F2     + 1.000 E5
                   .067
                 11.411
GEOMETRY=V6  =     .616*F2     + 1.000 E6
                   .069
                  8.942
```

```
COVARIANCES AMONG INDEPENDENT VARIABLES
 ---------------------------------------
I F2  -   F2                 .597*I
I F1  -   F1                 .072 I
                            8.308
```

M0, Single factor model

Single-factor model with Spearman's data (1904)

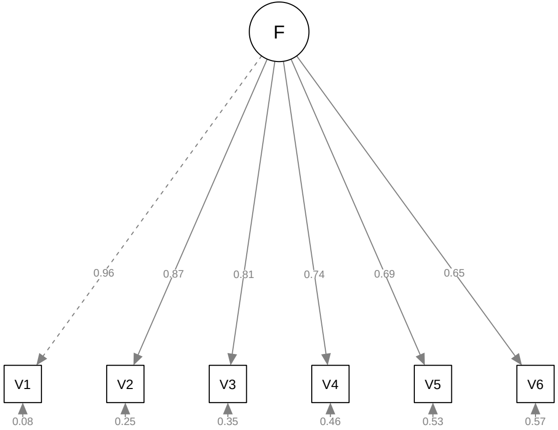
```
lower<-"1
  .83 1
  .78 .67 1
  .70 .64 .64 1
  .66 .65 .54 .45 1
   .63 .57 .51 .51 .40 1 "
  cova <- getCov(lower, names = c("V1", "V2", "V3", "V4", "V5",
  "V6"))
  # cova

     fit <- sem("F  =~   V1+V2+V3+V4+V5+V6 ", sample.cov = cova, sample.nobs = 23)
```

Summary fit

```r
summary(fit)
```

```
lavaan 0.6.16 ended normally after 24 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        12

  Number of observations                            23

Model Test User Model:

  Test statistic                                 1.739
  Degrees of freedom                                 9
  P-value (Chi-square)                           0.995

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  F =~
    V1                1.000
    V2                0.902    0.132    6.805    0.000
    V3                0.840    0.147    5.722    0.000
    V4                0.766    0.162    4.731    0.000
    V5                0.716    0.171    4.197    0.000
    V6                0.680    0.177    3.852    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    V1                0.075    0.060    1.253    0.211
```

Path diagram of single-factor model



Fitting Lawley and Maxwell model

```
cova<-  as.matrix(read.table(tmp <- textConnection("
 1 .439  .410  .288  .329  .248
.439  1  .351 .354 .320 .329
.410 .351 1 .164 .190 .181
.288 .354 .164 1 .595 .470
.329 .320 .190 .595 1 .464
.248 .329 .181 .470 .464 1
")))
close(tmp)

  # cova

fit <- sem("F  =~   V1+V2+V3+V4+V5+V6 ", sample.cov = cova, sample.nobs = 220)

     fit <- sem("F1  =~   V1+V2+V3; F2=~V4+V5+V6; F1 ~~ F2 ", sample.cov = cova, sample.nobs = 220)
```

Fitting Lawley and Maxwell model

```
cova<-  as.matrix(read.table(tmp <- textConnection("
 1 .439  .410  .288  .329  .248
.439  1  .351 .354 .320 .329
.410 .351 1 .164 .190 .181
.288 .354 .164 1 .595 .470
.329 .320 .190 .595 1 .464
.248 .329 .181 .470 .464 1
")))
close(tmp)

  # cova

fit <- sem("F  =~   V1+V2+V3+V4+V5+V6 ", sample.cov = cova, sample.nobs = 220)

     fit <- sem("F1  =~   V1+V2+V3; F2=~V4+V5+V6; F1 ~~ F2 ", sample.cov = cova, sample.nobs = 220)
```

summary fit of Lawley and Maxwell model

```r
summary(fit)
```

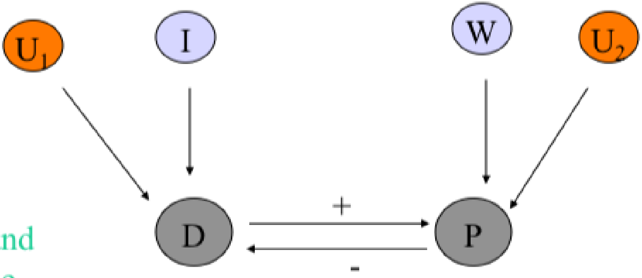
```
lavaan 0.6.16 ended normally after 24 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        13

  Number of observations                           220

Model Test User Model:

  Test statistic                                 7.990
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.434

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  F1 =~
    V1                1.000
    V2                0.979    0.152    6.427    0.000
    V3                0.776    0.134    5.809    0.000
  F2 =~
    V4                1.000
    V5                1.002    0.115    8.716    0.000
    V6                0.803    0.103    7.801    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
```

Section 4

Simultaneous equations (reverse causation?)

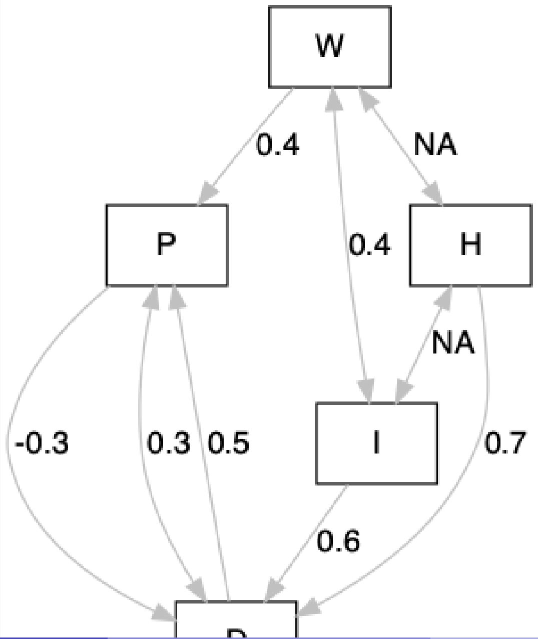
Simultaneous equations (reverse causation?)

Causal model with reciprocal effects



P = price
D = demand
I = Income
W = Wages

SEM with reverse causation



Sample data

```
      D     P     I     W     H
1  0.75  0.86 -0.86  0.94 -0.44
2 -1.23  0.55 -0.06  1.12  0.04
3  0.04 -1.54  0.72  0.91 -0.64
4  1.83  0.12  1.42 -0.06  1.24
5  0.13 -1.14 -1.19 -1.18  1.33
6  0.43  0.27  0.00  0.01  0.81
```

```
[1] 868   5
```

Separate vs. simultaneous regressions

Researchers may assume that those with a high body mass index (BMI) are more likely to be depressed when, in actuality, they find that depression leads to a high BMI. In reverse causality, the outcome precedes the cause, or the dependent variable precedes the regressor. With observational data, it is hard to evaluate whether the causal effect is in one direction or the contrary. In SEM, we can specify simultaneous effects to disentangle the direction of the causality issue. This calls for simultaneous regressions.

Single regression of D on P and I

	lhs	op	rhs	est	se	z	pvalue
1	D	~	P	0.416	0.027	15.309	0
2	D	~	I	0.300	0.034	8.873	0
3	D	~~	D	0.881	0.042	20.833	0
4	P	~~	P	1.552	0.000	NA	NA
5	P	~~	I	0.420	0.000	NA	NA
6	I	~~	I	1.002	0.000	NA	NA

Single regression of P on D and W

	lhs	op	rhs	est	se	z	pvalue
1	P	~	D	0.542	0.029	18.709	0
2	P	~	W	0.384	0.034	11.225	0
3	P	~~	P	0.969	0.046	20.833	0
4	D	~~	D	1.344	0.000	NA	NA
5	D	~~	W	0.112	0.000	NA	NA
6	W	~~	W	0.964	0.000	NA	NA

Simultaneous equations (D ∼ P + I and P∼ D+W)

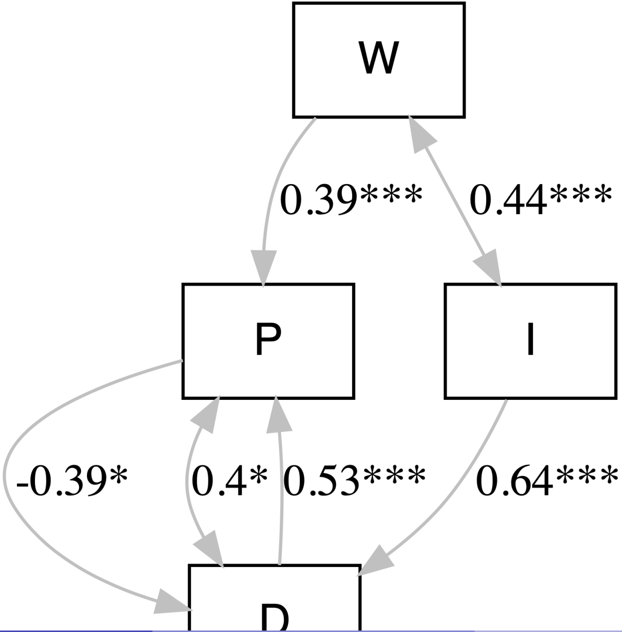
	lhs	op	rhs	est	se	z	pvalue
1	D	~	P	-0.134	0.083	-1.603	0.109
2	D	~	I	0.530	0.054	9.746	0.000
3	P	~	D	0.632	0.061	10.324	0.000
4	P	~	W	0.373	0.036	10.402	0.000
5	D	~~	D	1.296	0.140	9.223	0.000
6	P	~~	P	0.980	0.049	19.874	0.000
7	I	~~	I	1.002	0.000	NA	NA
8	I	~~	W	0.439	0.000	NA	NA
9	W	~~	W	0.964	0.000	NA	NA

	lhs	op	rhs	mi	epc	sepc.all	delta	ncp	power	decisior
11	D	~	W	5.954	-0.142	-0.120	0.1	2.971	0.407	**(m)**
10	D	~~	P	5.954	0.372	0.330	0.1	0.431	0.101	**(m)**
12	P	~	I	5.954	-0.152	-0.121	0.1	2.577	0.362	**(m)**

Modified model (cova of P and D)

	lhs	op	rhs	est	se	z	pvalue
1	D	~	P	-0.388	0.161	-2.414	0.016
2	D	~	I	0.637	0.081	7.856	0.000
3	P	~	D	0.528	0.071	7.484	0.000
4	P	~	W	0.385	0.035	11.009	0.000
5	D	~~	P	0.398	0.178	2.230	0.026
6	D	~~	D	1.769	0.365	4.844	0.000
7	P	~~	P	0.969	0.047	20.801	0.000
8	I	~~	I	1.002	0.000	NA	NA
9	I	~~	W	0.439	0.000	NA	NA
10	W	~~	W	0.964	0.000	NA	NA

path diagram of final model



Section 5

Foundations of SEM

Sewall Wright (1934, The Annals of Mathematical Statistics)

The absence of elasticity of supply in the case of potatoes applies only within a single year. The fact that the supply is strongly correlated with the price of the preceding year +.651 indicates that in the long run there is considerable elasticity. The method of path coefficients readily lends itself to deduction of this long time elasticity.

Let $\bar{P}$, $\bar{Q}$, $\bar{A}$ and $\bar{B}$ be the hypothetical averages of P, Q, A and B respectively over an indefinite (n) period of years. The problem is to deduce the elasticities toward which the long time supply and demand curves tend, from knowledge merely of the correlations from year to year. The following equation can be written from figure 32, where a, b, c and g are path coefficients pertaining to the paths indicated.

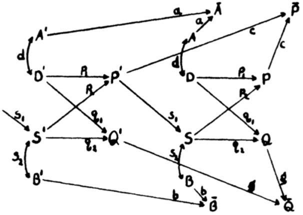


FIG. 32

[7] In two other cases studied by this method (P. G. Wright 1928) very different results were obtained. In the case of butter, the elasticity of supply came out 1.43, of demand —.62. In the case of flax seed, the elasticity of supply came out even greater, 2.39, while that of demand was —.80. But these are cases in which a high elasticity of supply is to be expected on a priori grounds. It is interesting to note that in cases in which it seems

Cowles Commission for research in economics

HISTORY OF THE COWLES COMMISSION 1932–1952

BY CARL F. CHRIST*

I. The founding of the Cowles Commission

The Cowles Commission for Research in Economics was founded in 1932. Alfred Cowles, president of Cowles and Company, an investment counseling firm in Colorado Springs, Colorado, initiated some inquiries into the accuracy of professional stock market forecasters over the period 1928–1932. This aroused his interest in fundamental economic research, which led him to offer his financial support toward the establishment of the Cowles Commission and to bear a significant share of the burden each year. Fortunately at the outset he encountered Harold T. Davis, a professor of mathematics at Indiana University

Cowles Commission (Koopmans and Hood; Trygve Haavelmo)

Koopmans and Hood

In Chapter VI of the Cowles Commission for Research in Economics Monograph No. 14,
The estimation of simultaneous linear economic relationships (Koopmans and Hood, 1953, p. 116-117)
. . . behavior equations:

$$h_1(\eta_t, \eta_{t-1}, \ldots, \eta_{t-s}; \zeta_t, \zeta_{t-1}, \ldots, \zeta_{t-s}; u_{1t}; \alpha_1) = 0$$

$$\ldots$$

$$h_G(\eta_t, \eta_{t-1}, \ldots, \eta_{t-s}; \zeta_t, \zeta_{t-1}, \ldots, \zeta_{t-s}; u_{Gt}; \alpha_G) = 0$$

$(t = 1, \ldots, T)$. Here $h_g (g = 1, \ldots, G)$ denote given scalar functions of the variables in parentheses, and the symbols α_g $(g = 1, \ldots, G)$ denote vectors of unknown behaviour parameters (elasticities of supply or demand), assumed to be independent [constant] of t
The behaviour equations are written in terms of the "true" endogeneous and exogeneous variables, whic are connected with the observed variables

. . . errors of observations

Koopmans and Hood (1953), p. 117: That errors of observations are disregarded in this chapter does not imply an a priori judgment that such errors are less important, in their effects on the choice of estimates and on the quality of these estimates, than diturbances in economic behaviour. footnote 5: It might be thougth that with gradual improvement in the methods of data collection, errors of observation would after a lapse of time be less important than the random elements intrinsic to economic behaviour. However, as Reiersol as pointyed out to one of the authors, as observation improve in accuracy and coverage, it will be possible to introduce more explanatory variables in each equation, thus reducing the variance of "unexplained" disturbances in behavior. [. . .] they most be regarded as an empirical question, to be setttled by methods of inference based on models recognizing errors of observation as well as disturbances in behaviour. The emphasis on disturbances in this and other chapters of this volume must be regarded rather as matter of tactics. "Shock-error models" are complicated. Complicated?, . . . not anymore, after the work of Karl G. Joreskog, to be commented below.

Joreskog's SEM approach

SEM (LISREL)

Jöreskog, K. G. (1970, ...) develop ML estimation and testing for a general shock-error-latent variable model + producing (with Dag Sörbom) the software LISREL to serve practitioners.
An exact relation $\eta = B\eta$ is contaminated by shocks

$$z = \Lambda\eta + \epsilon$$

$$\eta = B\eta + \zeta$$

with $\Psi := E\epsilon\epsilon'$ and $\Phi := E\zeta\zeta'$. Denote $\xi \equiv \Lambda(I - B)^{-1}\zeta$; then, we can write:

$$z = \Lambda(I - B)^{-1}\zeta + \epsilon = \xi + \epsilon$$

The matrices B, Λ, Ψ and Φ are functions of θ, the fundamental parameters of the model. The moment structure for the observable vector z is

$$\Sigma_{zz} = \Lambda(I - B)^{-1}\Phi(\Lambda(I - B)^{-1})^T + \Psi = \Sigma_{zz}(\theta)$$

where θ is the vector of free parameters of the coefficient matrices. [4]

[4]Proprietary software: LISREL, EQS, Mplus, CALIS, sem of Stata, AMOS, ... Free

K. G. Jöreskog's LISREL: the SEM approach

LISREL (SEM):

1. (a unifying) general "shocks-errors-latent variables" variable model. It encompasses regression, simultaneous equations, factor analysis, and combinations of the three.
2. ML estimation and testing of the general model, multiple group, robust se and test statistics (applicable to any subfamily of models)
3. Software for routinary practitioners use (not necessarily statisticians/econometricians) Nowadays: LISREL, EQS, MPlus, sem of Stata, sem and lavaan of free software R, ... LISREL was pioneering in the 70s.

A unifying tool for comparative empirical research. As in classical OLS regression, a variety of SEM software producing identical numerical results on a variety of models.

SEM: Estimation

Let S be the covariance matrix of the observables variables, Σ the population probability limit of S, θ the vector that collects the independent parameters of the model, and $\Sigma = \Sigma(\theta)$ the covariance structure function implied by the model.
The estimator $\hat{\theta}$ is the minimizer of a discrepancy function $F = F(S, \Sigma)$ of S and $\Sigma = \Sigma(\theta)$. The weighted least squares (WLS) and ML discrepancy functions are

$$F_{WLS}(\theta) = (s - \sigma)' V (s - \sigma)$$

and

$$F_{ML}(S, \Sigma(\theta)) = \ln \mid \Sigma(\theta) S^{-1} \mid + \text{tr} \{ S \Sigma(\theta)^{-1} \} - p$$

where p is the number of observed variables. Here s and σ are the vectors of non-redundant elements of the matrices S and Σ and $V > 0$, a weight matrix. [4]

SEM approach
Asymptotics

- $\text{avar}(\hat{\theta}) = (\Delta' V \Delta)^{-1} \Delta' V \Gamma V \Delta (\Delta' V \Delta)^{-1}$
  when $V \Gamma V = V$, then $\text{avar}(\hat{\theta}) = (\Delta' V \Delta)^{-1}$
- When the model holds: $T = n \times \hat{F} \sim \chi^2_r$,
  r is difference among the number of distinct moments and the number of independent parameters (the so-called model degrees of freedom)
- LM (Score tests) and Wald test statistics are available to assist in model modification
- Scaled Chi-square, $T_S \equiv \frac{1}{c} T_{ML}$, where $c := \text{tr}(U\Gamma)/\text{df}$,
  $U = V - V\Delta(\Delta' V \Delta)^{-1} \Delta' V$ and df is model degrees of freedom. [5]

It is assumed $\sqrt{n}(s - \sigma) \xrightarrow{D} \text{N}(0, \Gamma)$, and we let $\Delta = \partial\sigma / \partial\theta'$ and $V = \partial F / \partial\sigma\partial\sigma'$. [6]

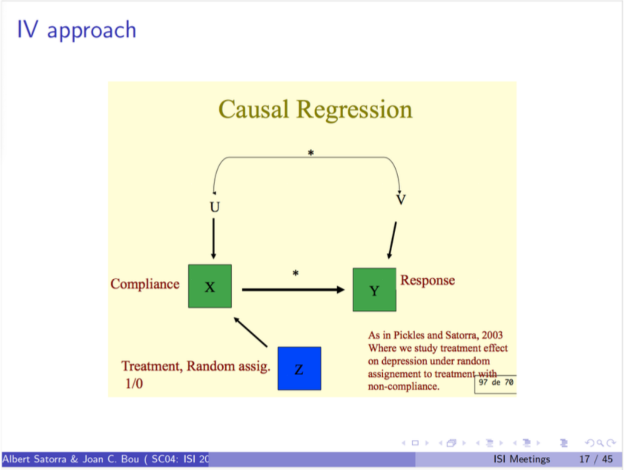
[5] This is the so-called Satorra-Bentler scaled test in `sem` of `Stata`. See https://www.youtube.com/watch?v=-wtHh3CiWlw

[6] For more details on the asymptotics, see Satorra, A. (2002). Asymptotic Robustness

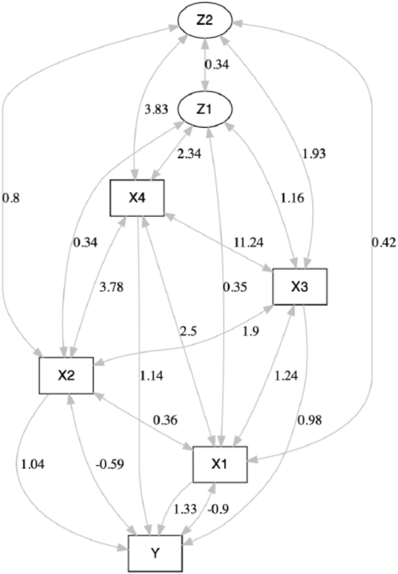
Section 6

Models: semIV, MTMM, MIMIC, ...

Not so recent, but unpublished, Pickles and Satorra (2003)



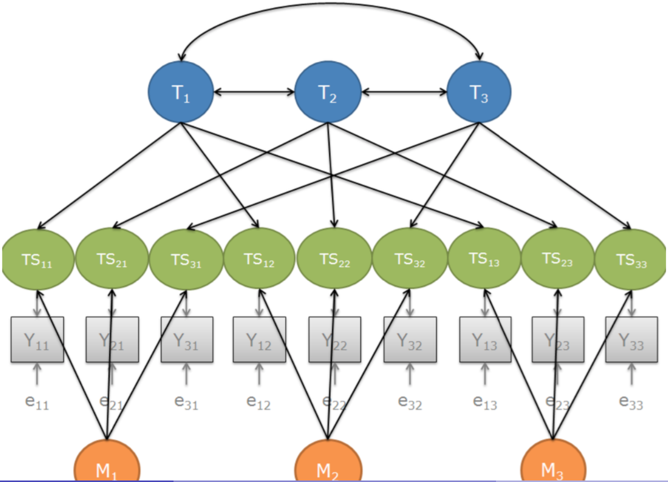
Regression with many IVs



Measurement Models, Saris et. al.

MTMM: True Score Model

Saris and Andrews (1991)



Kenny's

CT-CM - MTMM

https://davidakenny.net/cm/mtmm.htm

Example

Mount (1984) presented ratings of managers on Administration, Feedback, and Consideration by the managers' supervisors, the managers themse

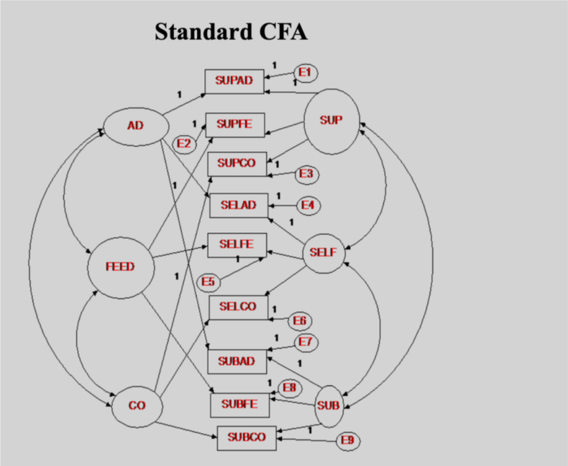
		Supervisor			Self			Subordinate		
		A	F	C	A	F	C	A	F	C
Supervisor										
	A	1.00								
	F	.35	1.00							
	C	.10	.38	1.00						
Self										
	A	**.56**	.17	.04	1.00					
	F	.20	**.26**	.18	.33	1.00				
	C	-.01	-.03	**.35**	.10	.16	1.00			
Subordinate										
	A	**.32**	.17	.20	**.27**	.26	-.02	1.00		
	F	-.03	**.07**	.28	.01	**.17**	.14	.26	1.00	
	A	-.10	.14	**.49**	.00	.05	**.40**	.17	.52	1.00

bold correlations: validity diagonal

See David Kenny's example

CT-CM - MTMM

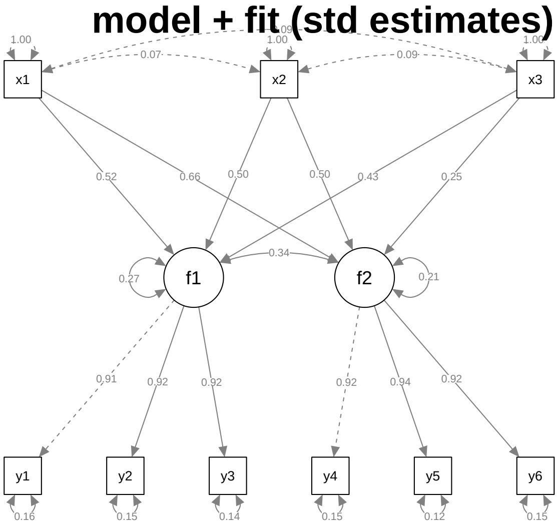
https://davidakenny.net/cm/mtmm.htm



MIMIC and its Sintaxis

```
# Example 5.8 from mplus user guide:
Data <- read.table("http://www.statmodel.com/usersguide/chap5/
names(Data) <- c(paste("y", 1:6, sep=""),
                 paste("x", 1:3, sep=""))

# Model:
model.Lavaan <- 'f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f1 + f2 ~ x1 + x2 + x3 '
```

Path diagram of MIMIC



MIMIC: tabacco issp93 data

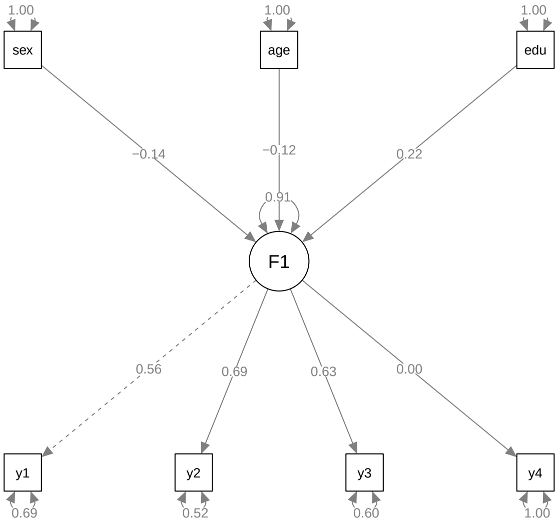
See also stata manual on SEM stata manual on SEM Variables:

```
1. Full text of y1: We believe too often in science, and not enough in
            feelings and faith.

2.  Full text of y2: Overall, modern science does more harm than good.

3.  Full text of y3: Any change humans cause in nature, no matter how
            scientific, is likely to make things worse.

4.  Full text of y4: Modern science will solve our environmental problems
            with little change to our way of life.
```

MIMIC: tabacco issp93 data (data summary and model)

```
   lhs op rhs    est    se      z pvalue std.all
1   F1 =~  y1  1.000 0.000     NA     NA   0.556
2   F1 =~  y2  1.331 0.124 10.693  0.000   0.692
3   F1 =~  y3  1.162 0.107 10.830  0.000   0.629
4   F1 =~  y4  0.008 0.080  0.097  0.923   0.004
5   F1  ~ sex -0.167 0.051 -3.284  0.001  -0.135
6   F1  ~ age -0.044 0.016 -2.788  0.005  -0.116
7   F1  ~ edu  0.104 0.021  4.985  0.000   0.217
8   y1 ~~  y1  0.852 0.053 16.098  0.000   0.691
9   y2 ~~  y2  0.737 0.067 10.978  0.000   0.522
10  y3 ~~  y3  0.789 0.058 13.580  0.000   0.605
11  y4 ~~  y4  1.454 0.070 20.868  0.000   1.000
12  F1 ~~  F1  0.345 0.050  6.956  0.000   0.905
13 sex ~~ sex  0.250 0.000     NA     NA   1.000
14 sex ~~ age  0.002 0.000     NA     NA   0.003
15 sex ~~ edu -0.059 0.000     NA     NA  -0.092
16 age ~~ age  2.622 0.000     NA     NA   1.000
17 age ~~ edu -0.447 0.000     NA     NA  -0.214
18 edu ~~ edu  1.661 0.000     NA     NA   1.000

 chisq     df pvalue  rmsea
 24.40  11.00   0.01   0.04
```

MIMIC: tabacco issp93 data (model)

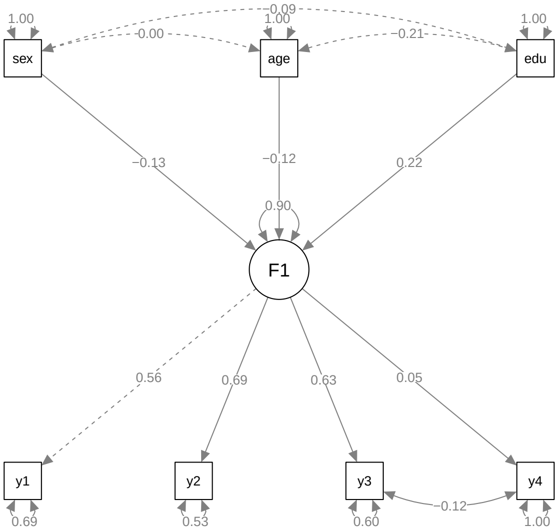


modified model, adding y3 $\sim\sim$ y4

```
model2<-paste(model1," y3 ~~  y4", sep=";")
fit2 <- sem(model2, data=d)
#summary(fit2)
parameterestimates(fit2, ci=FALSE)
```

```
   lhs op rhs    est    se      z pvalue
1   F1 =~  y1  1.000 0.000     NA     NA
2   F1 =~  y2  1.318 0.122 10.793  0.000
3   F1 =~  y3  1.162 0.107 10.837  0.000
4   F1 =~  y4  0.098 0.086  1.142  0.254
5   F1  ~ sex -0.162 0.051 -3.183  0.001
6   F1  ~ age -0.045 0.016 -2.851  0.004
7   F1  ~ edu  0.107 0.021  5.110  0.000
8   y3 ~~  y4 -0.128 0.044 -2.933  0.003
9   y1 ~~  y1  0.849 0.053 16.106  0.000
10  y2 ~~  y2  0.746 0.066 11.259  0.000
11  y3 ~~  y3  0.786 0.058 13.522  0.000
12  y4 ~~  y4  1.450 0.070 20.832  0.000
13  F1 ~~  F1  0.346 0.050  6.995  0.000
14 sex ~~ sex  0.250 0.000     NA     NA
15 sex ~~ age  0.002 0.000     NA     NA
16 sex ~~ edu -0.059 0.000     NA     NA
17 age ~~ age  2.622 0.000     NA     NA
18 age ~~ edu -0.447 0.000     NA     NA
19 edu ~~ edu  1.661 0.000     NA     NA
```

Final modified model



No continuous variables: tetrachoric, polychorical, and poliserial correlations

```r
fit <- sem(model2, data=d, ordered=names(d)[2:5])

# [1] "y1" "y2" "y3" "y4"

summary(fit)
```

Using tetrachoric, polychorical, and poliserial correlations

```r
fit <- sem(model2, data=d, ordered= names(d)[2:5])
parameterestimates(fit, ci=FALSE)
```

```
   lhs  op rhs    est    se      z pvalue
1   F1  =~  y1  1.000 0.000     NA     NA
2   F1  =~  y2  1.250 0.095 13.150  0.000
3   F1  =~  y3  1.149 0.085 13.554  0.000
4   F1  =~  y4  0.037 0.071  0.523  0.601
5   F1   ~ sex -0.166 0.049 -3.379  0.001
6   F1   ~ age -0.041 0.015 -2.736  0.006
7   F1   ~ edu  0.098 0.020  4.934  0.000
8   y3  ~~  y4 -0.102 0.028 -3.625  0.000
9   y1   |  t1 -1.374 0.188 -7.312  0.000
10  y1   |  t2 -0.242 0.183 -1.319  0.187
11  y1   |  t3  0.398 0.185  2.156  0.031
12  y1   |  t4  1.368 0.197  6.935  0.000
13  y2   |  t1 -1.431 0.191 -7.501  0.000
14  y2   |  t2 -0.593 0.184 -3.232  0.001
15  y2   |  t3  0.048 0.183  0.263  0.792
16  y2   |  t4  1.026 0.187  5.485  0.000
17  y3   |  t1 -1.090 0.183 -5.962  0.000
18  y3   |  t2 -0.042 0.178 -0.236  0.813
19  y3   |  t3  0.593 0.181  3.282  0.001
20  y3   |  t4  1.447 0.194  7.465  0.000
21  y4   |  t1 -1.198 0.186 -6.434  0.000
22  y4   |  t2 -0.135 0.174 -0.779  0.436
23  y4   |  t3  0.464 0.174  2.672  0.008
24  y4   |  t4  1.243 0.177  7.040  0.000
25  y1  ~~  y1  0.676 0.000     NA     NA
26  y2  ~~  y2  0.494 0.000     NA     NA
27  y3  ~~  y3  0.573 0.000     NA     NA
28  y4  ~~  y4  1.000 0.000     NA     NA
29  F1  ~~  F1  0.324 0.037  8.722  0.000
30 sex  ~~ sex  0.250 0.000     NA     NA
31 sex  ~~ age  0.002 0.000     NA     NA
```

Examples of MIMIC in health research

> J Health Econ. 1987 Mar;6(1):27-42. doi: 10.1016/0167-6296(87)90029-4.

Health status estimation on the basis of MIMIC-health care models

R C Van Vliet, B M Van Praag

PMID: 10282728 DOI: 10.1016/0167-6296(87)90029-4

Abstract

In this paper we propose a new method for deriving health indexes from MIMIC-health care models. This method differs from the traditional approach in that the health indexes are not based on the causes of health but on transformations of the health indicators. These transformations are employed mainly to correct for the effects of variables which do influence the health indicators but not health status, H*, itself, like availability of medical specialists. The method is applied to a MIMIC-health care model, which is estimated on a Dutch database. The estimated parameters of this model and the derived health indexes may be used in future research to collect only those health indicators and related variables which appear to contain relevant information on H*.

Similar articles

Health as an unobservable: a MIMIC-model of demand for health care.
Van de Ven WP, Van der Gaag J.
J Health Econ. 1982 Aug;1(2):157-83. doi: 10.1016/0167-6296(82)90013-3.
PMID: 10263954

Health indexes sensitive to medical care variation.
Martini CJ, Allan GH, Davison J, Backett EM.

Compact model expression

Y and X are response and treatment variables resp., W and Z the time-varying and time-invariant covariates. To simplify, take $T = 3$ with single variables o Y, X, W and Z at each time point. Generalitzation to models involving more time points and variables, are straightforward.[1]

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ * & 0 & 0 \\ 0 & * & 0 \end{pmatrix} y + \begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{pmatrix} x + \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix} w + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \gamma_Z + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \eta + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mu + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

$$y = B\, y + A\, x + \Gamma_w\, w + \Gamma_Z\, \gamma_Z + 1_T\, \eta + I_T\, \mu + I_T\, \epsilon \tag{2}$$

[1] The dynamics may induce an initial condition equation $Y_1 = \eta + \mu_1 + \epsilon_1$ with variables X_1 and W_1 fully suppressed from the analysis. Default is no initial condition.

Moment matrices of independent variables, vector ζ

$$\Phi = \begin{pmatrix} & \epsilon & x & w & Z & \eta & 1 \\ \epsilon & \Omega_{\epsilon,\epsilon} & \Omega_{\epsilon,x} & 0 & 0 & 0 & 0 \\ x & \Omega_{x,\epsilon} & * & * & * & * & 0 \\ w & 0 & * & * & * & * & 0 \\ Z & 0 & * & * & * & * & 0 \\ \eta & 0 & * & * & * & * & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

where the matrix $\Omega_{\epsilon,\epsilon}$ is a diagonal matrix with free parameters in the diagonal. The matrix $\Omega_{x,\epsilon}$ takes care of sequential endogeneity.
\end{frame}

\begin{frame}{ Strict exogeneity vs Sequential exogeneity:} {Chamberlain (1992)} [2]

{Strict exogeneity:}

$$\begin{pmatrix} & X_1 & X_2 & X_3 \\ \epsilon_1 & 0 & 0 & 0 \\ \epsilon_2 & 0 & 0 & 0 \\ \epsilon_3 & 0 & 0 & 0 \end{pmatrix} \quad (4)$$

too stringent in many applications.

{Sequential exogeneity:}

$$\begin{pmatrix} & X_1 & X_2 & X_3 \\ \epsilon_1 & 0 & * & * \\ \epsilon_2 & 0 & 0 & * \\ \epsilon_3 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

Sequential exogeneity allows covariation of the ϵ_t with the $X_{t+h},\ h > 0$; i.e., it allows for the so-called {endogeneity in panel data}.

[2] Chamberlain, G. (1992). Comment: sequential moment restrictions in panel data. *Journal of Business and Economic Statistics, 10*(1), 20-26.

path diagram for dynamic model

Fixed effects (FE) panel data with: dynamics, endogeneity, and lagged effects, the FE-PDELE model

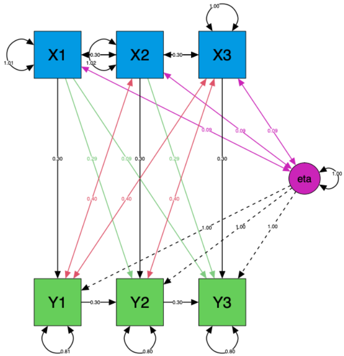
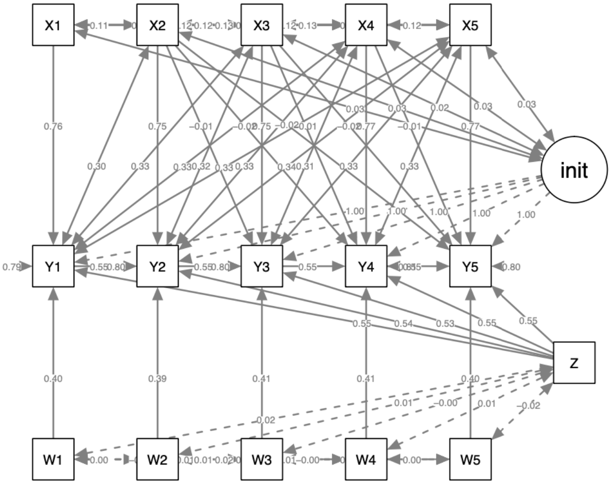


Figure: Path diagram for FE panel data with dynamics, endogeneity and lagged effects The true value of X on Y is 0.3, it is estimated as 0.305 (se=0.034). FE model assumes that the latent omitted effects of the model can be arbitrarily correlated with the included variables; the alternative RE-PDELE is obtained suppressing the correlations of η with the Xs.

Path diagram Dynamic, Endogeneity and Lagged Effects

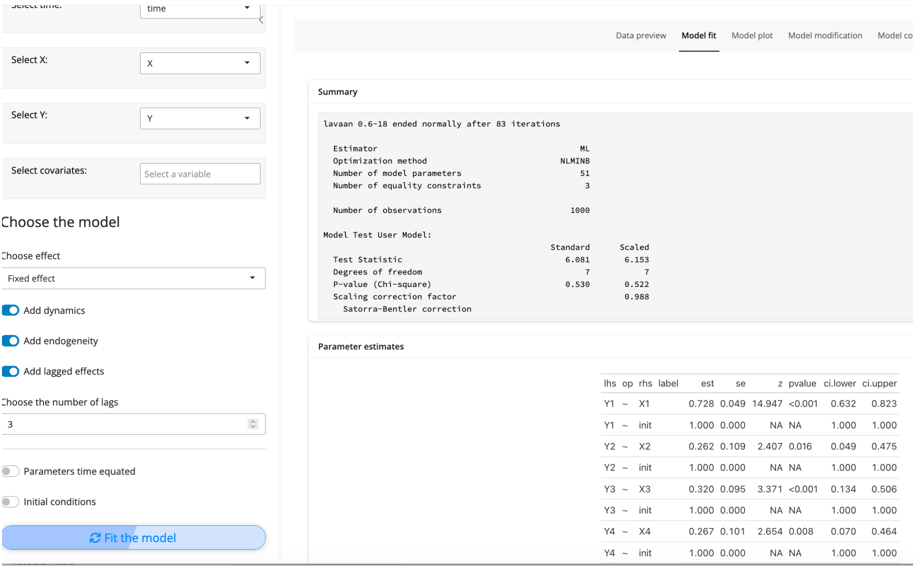


Dynamic Panel Data analysis, a shiny application (in development)

Thi shiny application is work of

Pau Satorra
web: https://pasahe.github.io/PauSatorra
Research Institute and Hospital (IGTP),
Badalona, Barcelona

Model specification and sintaxis (changed, refreshed)



lhs	op	rhs	label	est	se	z	pvalue	ci.lower	ci.upper
Y1	~	X1		0.728	0.049	14.947	<0.001	0.632	0.823
Y1	~	init		1.000	0.000	NA	NA	1.000	1.000
Y2	~	X2		0.262	0.109	2.407	0.016	0.049	0.475
Y2	~	init		1.000	0.000	NA	NA	1.000	1.000
Y3	~	X3		0.320	0.095	3.371	<0.001	0.134	0.506
Y3	~	init		1.000	0.000	NA	NA	1.000	1.000
Y4	~	X4		0.267	0.101	2.654	0.008	0.070	0.464
Y4	~	init		1.000	0.000	NA	NA	1.000	1.000

Figure 25: Panel Data

The path diagram

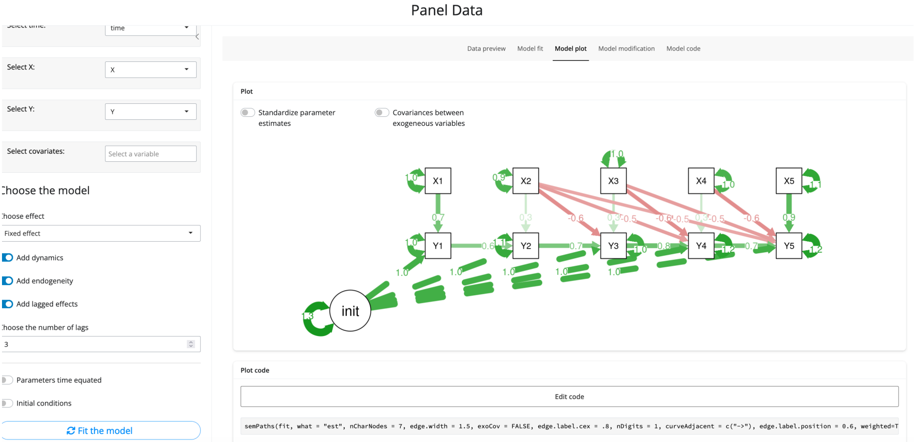


Figure 26: Panel Data

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The End

Thank You!

```
[1] "fiml on dd:"

  lhs op rhs    est    se       z pvalue ci.lower ci.upper
1   y  ~  x1 -4.432 0.372 -11.900      0   -5.162   -3.702
2   y  ~  x2  1.910 0.111  17.189      0    1.693    2.128
3   y ~~   y  0.777 0.061  12.728      0    0.658    0.897
4  x1 ~~  x1  0.021 0.000      NA     NA    0.021    0.021
5  x1 ~~  x2  0.023 0.000      NA     NA    0.023    0.023
6  x2 ~~  x2  0.265 0.000      NA     NA    0.265    0.265
7   y ~1      2.073 0.134  15.428      0    1.810    2.337
8  x1 ~1      0.215 0.000      NA     NA    0.215    0.215
9  x2 ~1      1.268 0.000      NA     NA    1.268    1.268

[1] "lm() on dd:"

   Estimate Std. Error  t value Pr(>|t|)
x1  -4.4324     0.3742 -11.8449        0
x2   1.9104     0.1117  17.1095        0
```